SUSAN B. TABER

Using *Alice in Wonderland* to Teach Multiplication of Fractions

ALICE'S ADVENTURES IN WONDERLAND BY Lewis Carroll is a captivating story that appeals to students in the middle grades. It also provides an opportunity to help students develop an understanding of some complex mathematical content, particularly the multiplication of fractions, which is introduced in the middle grades.

First published in 1865, *Alice's Adventures in Wonderland* began as a story told by Charles Lutwidge Dodgson, an Oxford don and lecturer in mathematics, to the three Liddell sisters, ages 8 to 13, during an afternoon's boat trip in 1862. The story might have vanished into the summer's air but for Alice Liddell, age 10, who begged Dodgson to write it down. The first version, *Alice's Adventures Underground*, was handwritten and illustrated by Dodgson and presented to Alice Liddell as a Christmas gift in 1864. For publication, Dodgson revised and expanded the story, which was then illustrated by John Tenniel. He also changed the name to *Alice's Adventures in Wonderland*, and the book was published as the work of Lewis Carroll (Cohen 1995).

Alice's Adventures in Wonderland can be used as the focus of an interdisciplinary language arts and mathematics unit. The story has a Lexile rating of 890 and is considered to be at an appropriate reading level for students in grades 4 through 8 (Scho-

SUSAN TABER, *taber@rowan.edu, teaches undergraduate and graduate courses in mathematics pedagogy and curriculum at Rowan University, Glassboro, NJ 08028. In addition to studying students' understanding of mathematics and the effect of instruction on students' thinking, she frequently conducts professional development workshops and courses.*

lastic 2004). (*A Lexile rating is a scale used to determine the reading level of a particular text.—Ed.*) The story is familiar to most children who have either read the book or who have seen the movie; that familiarity can support students' understanding of the text. The following list describes four crucial mathematics concepts related to multiplication of fractions and proportional reasoning that can be developed during an interdisciplinary unit:

1. Multiplication by a number less than 1 results in a product that is less than the original number, and multiplication by a number greater than 1 results in a product that is more than the original number
2. Multiplication by a very small number will result in a very small product but will not result in a product of 0
3. The distinction between additive/subtractive change and multiplicative change
4. The multiplicative relationships that are at the heart of proportionality and similarity

Multiplication by a number less than 1. The multiplication of fractions is a complex mathematics concept and is often difficult for students *and* adults. Studies have shown that even after learning to compute the products of fractions or decimals, most students have difficulty solving multiplication word problems involving fractions or decimals less than 1 (Graeber 1993; Greer 1992). For example, after two weeks of instruction on multiplication of fractions, the majority of students in a fifth-grade class solved problems like the following by dividing the whole number by the denominator and multiplying the result by the numerator.

> JoAnne's dog weighs 21 pounds on earth. On a smaller planet it would weigh 2/3 as much. How much would it weigh on the other planet?

On the other hand, the students used multiplication to solve a problem like this one:

> Dave's science report was 1/3 of a page long. Frank's history report was 12 times as long as Dave's report. How many pages long was Frank's report?

Even though the students knew how to correctly find the product of two fractions and of whole numbers and fractions, most of them did not think of the first of these two problems as being a multiplication problem even though it was placed immediately after the science-report problem on the page.

One reason for this is that the two factors in multiplicative situations play different roles. One factor is the operator or multiplier; it affects the change in the other factor, the quantity. Although people easily recognize problems that have whole-number multipliers as being multiplication situations, difficulties arise when the multipliers are fractions or decimals less than 1. Many students and adults think that these situations do not involve multiplication but rather division or subtraction because the resulting quantity is smaller than the quantity named in the problem (Taber 1999). If students become accustomed to solving problems like the first one (JoAnne's dog) by dividing and then multiplying, they may come to believe that problems that ask them to find a fraction of a quantity are solved by division. If the problem asked them to find 2/3 of 21 1/2 pounds or of 5/8 pound, however, the method of dividing by the denominator and multiplying by the numerator would be much more difficult to carry out.

Instruction on multiplication of fractions, therefore, needs to help students extend and transform their understanding of whole-number multiplication to include new kinds of problem situations. For students to understand that multiplication can solve problems with fraction multipliers requires them to extend and reconceptualize their understanding of multiplication to include multiplication by fraction multipliers.

In teaching a unit on multiplication of fractions to another class of fifth-grade students, I used the story of *Alice's Adventures in Wonderland* to help students understand that multiplication can shrink quantities as well as enlarge them. The students and I reviewed the various things that happened to Alice after she ate cake or drank potions from bottles. "Did you know," I asked them, "that Lewis Carroll was really a mathematician named Charles L. Dodgson? Let's see how we can show with mathematics the kinds of things that happened to Alice in the story. Let's suppose that Alice is 4 1/2 feet tall, that's 54 inches." I wrote "54 inches" on the board and drew a little bottle next to it. "Now we're going to have her drink from this little bottle that will make her 1/9 of her present size," I said as I wrote 1/9 on the bottle. "How tall will she be after she drinks?"

"Six inches," the students replied. I wrote "=6" on the board to the right of the bottle.

"That's pretty small," I said. "Suppose it was a 1/3 bottle she drank from instead?" I wrote "54" and drew a bottle labeled 1/3 next to it. "How tall would she be then?"

"Eighteen inches," Jamal said, and the rest of the class nodded in agreement. I recorded "=18 inches" on the board.

"Let's let her drink from a . . ." I began, but was interrupted by Donald, who said, "One-sixth bottle."

I wrote "54" and a bottle labeled 1/6 on the board.

"That's 9 inches," Tamika said. I wrote "= 9 inches" on the board.

"Now I want to see what would happen if she drank from a 5/6 bottle," I said, writing 54 followed by a bottle labeled 5/6.

"Oh, this is easy," said several students. "It's 45 inches."

I asked Ken to explain how he found 45 inches. "If 1/6 is 9 inches," he said, "then 5/6 would be 9 times 5 and that's 45."

I called the students' attention to the list we had made on the board (**fig. 1**). "This shows that we're changing Alice's height when she drinks from the bottles. What mathematical symbol should we put in here between Alice's height and the bottles?"

Alice's Height

Start	Bottle	Finish
54 inches	1/9	= 6 inches
54 inches	1/3	= 18 inches
54 inches	1/6	= 9 inches
54 inches	5/6	= 45 inches

Fig. 1 Changes in Alice's size

ILLUSTRATION BY JOHN TENNIEL, FROM *ALICE'S ADVENTURES IN WONDERLAND* BY LEWIS CARROLL

Fig. 2 Alice and the Caterpillar

"Division," Jamal said. "You're dividing 54 by 1/9."

Stephanie raised her hand. "It's multiplication. You're multiplying 54 by 1/9."

"How many think we're dividing?" I asked. About a third of the students raised their hands. "How many think we're multiplying?" Another third raised their hands. "How many are not sure? Let's take a small Alice and make her get bigger. Maybe that will help us figure out what's going on." We then pretended that Alice ate cakes labeled with various whole numbers and fractions greater than 1. Students computed her height after she ate each cake. We then discussed the appropriate operation symbol to put between the beginning height and either the cake or the bottle. Students agreed that the multiplication symbol would work in either situation: when Alice was shrinking as well as when she was growing.

Before beginning the lesson the next day, one student's question indicated to me that the students were beginning to understand that multiplication could indeed decrease a quantity. Carrie asked, "Why is it when we times 29 times 2/9 that the answer goes down?"

I wanted to be sure that I understood her question, so I asked one of my own: "Your question is that if we multiply 29 times 2/9, it's smaller than what?"

"Twenty-nine," Carrie replied. I asked if anyone in the class could think of a reason.

Nina quickly raised her hand, "If you times 29 times 1, you get 29, but 2/9 isn't a complete 1, so you get less than 29."

Alan contributed, "Twenty-nine 2/9 is like counting 2/9 [a total of] 29 times."

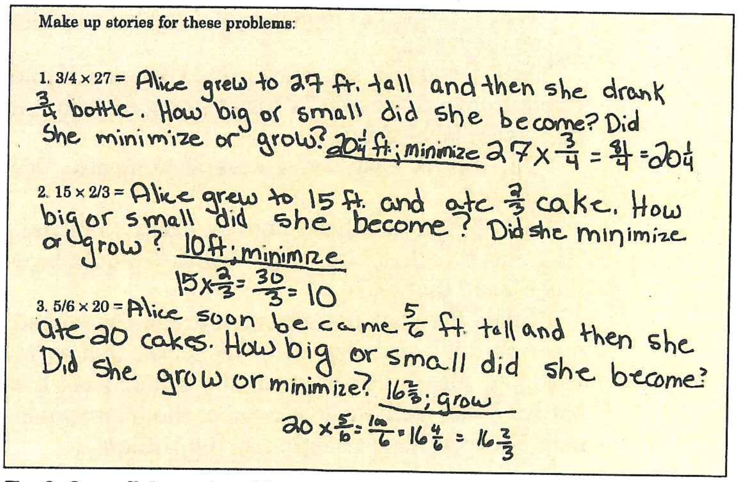
Carrie's question and Nina's and Alan's responses showed that students were beginning to understand the relationship between multiplying by numbers greater than 1 and less than 1. This principle of multiplication is articulated by the Caterpillar in the story. Before meeting the Caterpillar, Alice is frustrated by her inability to control her changes in size. She has no reliable way of predicting whether she will grow larger or smaller. Drinking from the first bottle makes her shrink to 10 inches, but drinking from the bottle she finds in the White Rabbit's house makes her grow too large for the house. Eating one cake enlarges her from 10 inches to over 9 feet tall, but eating the pebble-cakes causes her to shrink small enough so that she can walk through the rabbit's door. Alice begins to be able to control her changes in size after her conversation with the Caterpillar:

> "One side will make you grow taller, and the other side will make you grow shorter."
> "One side of *what*? The other side of *what*?" thought Alice to herself.
> "Of the mushroom," said the Caterpillar. (Carroll 1992, p. 68)

Although a round mushroom cap does not have "sides," Alice copes with the situation by breaking off diametrically opposite pieces of the mushroom. (See the illustration in **fig. 2**.) After experimenting to find out which part of the mushroom makes her grow and which makes her shrink, Alice is able to change her size to suit her purposes: to participate in the tea party and then to unlock the door and enter the garden. Alice's ability to control whether she will grow larger or smaller corresponds to students' understanding that multiplication by a number larger than 1 increases the quantity being multiplied, whereas multiplying by a number less than 1 decreases it.

The worksheet titled "How Tall Is Alice?" gave students a chance to practice multiplication of fractions and to explain their emerging understanding of the relationships between the magnitude of the multiplier and its effect on the starting quantity. The stories that Carmella wrote for the three problems at the end of the worksheet (**fig. 3**) show that she is developing an understanding that multiplication can shrink ("minimize") as well as enlarge ("grow") quantities. Although it is conventional to think of the first quantity in the multiplication expression as the multiplier, it is clear from Carmella's word problems that she is not following that convention. In the first, she describes finding 3/4 of 27, and in the second problem, she finds 2/3 of 15. Her third problem describes finding 20 times as much as 5/6. Nevertheless, her story problems indicate an understanding that multiplication can result in "shrinking" or "growing."

To compare changes in these students' thinking with those of the students in the first class about multiplication problems involving fractions, I gave the students a pretest and a posttest of twenty word problems, ten with fraction multipliers and ten with whole-number multipliers. I compared the solution strategies they used with those employed by the first class of fifth graders on the same pre- and posttests. **Table 1** shows that the students who had discussed *Alice's Adventures in Wonderland* and had learned to represent changes in her size with multiplication used multiplication much more frequently on all the posttest problems, whether the multipliers were whole numbers or fractions. Of the "Alice" students, 75 percent used multiplication to solve all the problems and only 19 percent used division to solve problems with fraction multipliers compared with 28 percent of the "non-Alice" students who used multiplication to solve all problems and 64 percent who divided to solve the posttest problems that had fraction multipliers. Many more "Alice" students had successfully extended their ideas about multiplication to include multiplication by fractional multipliers.


Make up stories for these problems:

1. 3/4 × 27 = Alice grew to 27 ft. tall and then she drank 3/4 bottle. How big or small did she become? Did she minimize or grow? 20 1/4 ft; minimize 27 × 3/4 = 81/4 = 20 1/4

2. 15 × 2/3 = Alice grew to 15 ft. and ate 2/3 cake. How big or small did she become? Did she minimize or grow? 10 ft; minimze 15 × 2/3 = 30/3 = 10

3. 5/6 × 20 = Alice soon became 5/6 ft. tall and then she ate 20 cakes. How big or small did she become? Did she grow or minimize? 16 2/3; grow 20 × 5/6 = 100/6 = 16 4/6 = 16 2/3

Fig. 3 Carmella's word problems

Interviews with five "Alice" students confirmed these changes in their understanding of multiplication. Before the instruction, they said that the only problems that could be solved by multiplication or repeated addition were those with whole-number

TABLE 1

Percent of Students Employing Each Strategy Pattern to Solve Word Problems with Whole-Number and Fraction Multipliers

	Non-Alice Students (n = 25)		Alice Students (n = 21)	
Strategy Pattern	Pretest	Posttest	Pretest	Posttest
Multiplied to solve all problems	4	28	0	77
Multiplied to solve problems with whole-number multipliers and divided to solve problems with fraction multipliers	32	60	62	19
Divided to solve all problems	12	4	5	0
Used a diagram	8	0	10	0
No strategy pattern	44	8	24	5

ILLUSTRATION BY JOHN TENNIEL, FROM *ALICE'S ADVENTURES IN WONDERLAND* BY LEWIS CARROLL

Fig. 4 The Cheshire Cat

multipliers. Students who were able to solve the problems with fraction multipliers used division strategies. See the following problem:

> A carton will hold 42 notebooks. If the carton is 5/6 full, how many notebooks are in the carton?

After instruction, Tamika, like the other interviewed students, said that she would solve the problem by multiplying 42 by 5/6. "Because you're going to find it when it's in sixths and then you'll find the 5. When you times it and you're going to put it into sixths, it's just like dividing it into sixths and then you get the 5 sixths," she said. She wrote,

$$42 \times \frac{5}{6} = \frac{210}{6} = 35.$$

Her explanation indicates that she understood that 5/6 of a quantity could be found by multiplying by 5/6 and that she understood the composite nature of the multiplier 5/6; it simultaneously partitions the quantity into sixths and collects 5 of them.

Alice's changes in size thus provide students with the opportunity to develop an understanding of how multiplication by fractions can symbolize multiplicative situations in which the product is smaller than the original quantity. It can also extend their knowledge of whole-number multiplication to include multiplication by numbers less than 1. Other incidents in the story can afford opportunities to develop additional elegant, yet fundamental, concepts related to multiplication and proportionality.

Multiplication by a very small number. The idea of a limit or how small a product can be can also be discussed. In chapter 2, Alice discovers that fanning herself is causing her to shrink rapidly, and she drops the fan "just in time to save herself from shrinking away altogether" (Carroll 1992, p. 22). Alice fears that if she gets to be too small, she will cease to exist. Ask students to find Alice's height if it were multiplied by a very small fraction. Students might suggest multiplying by 1 one-millionth or 1 one-billionth. This discussion gives the class a chance to explore informally ideas such as whether there is a smallest fraction and whether Alice would ever go out of existence and completely vanish. Although Alice might become too small to see with the unaided eye, she would not vanish unless her height were multiplied by 0.

The distinction between additive/subtractive change and multiplicative change. Asking students to compare and contrast the way in which the Cheshire Cat appears and disappears with the changes in Alice's size provides an opportunity to discuss the distinctions between additive and multiplicative change. Whether Alice shrinks or grows taller, she is always a complete girl with head, hands, feet, and so on. The Cheshire Cat, however, appears and disappears part by part (see **fig. 4**). In chapter 6, the cat disappears bit by bit until only the grin remains. In chapter 8, the grin appears first, but Alice decides not to speak to the cat until its ears are seen. The Cheshire Cat can also disappear completely. His transformations from visibility to invisibility result from the additive operation of subtraction, in contrast to Alice's transformations, which are multiplicative. It is interesting to note that some characters in the story, such as the baby who becomes a pig and the playing cards, are transformed in ways that cannot be represented mathematically.

Multiplicative relationships at the heart of proportionality and similarity. Multiplicative change also underlies the concepts of proportionality and similarity in the context of geometric figures. Sometimes Alice's changes in size preserve her proportions, but other changes are disproportionate, as when she says she is "opening out like the largest telescope that ever was" (Carroll 1992, p. 15); the illustration in chapter 2 shows her as disproportionately taller (**fig. 5**). Students could further explore proportionality, similarity, and the distinction between additive and multiplicative change by comparing pairs of rectangles like those in **figure 6**, stating whether the relationship between the rectangles in each pair is additive or multiplicative and whether the rectangles are similar. The two rectangles in **figure 6a** are similar because for both pairs, the sides of one rectangle are 5/3 (or 3/5) the length of the sides of the other rectangle. The rectangles in **figure 6b** are not similar. The sides of the smaller rectangle are 2 units less than the sides of the larger rectangle.

ILLUSTRATION BY JOHN TENNIEL, FROM *ALICE'S ADVENTURES IN WONDERLAND* BY LEWIS CARROLL

Fig. 5 Alice grows out of proportion.

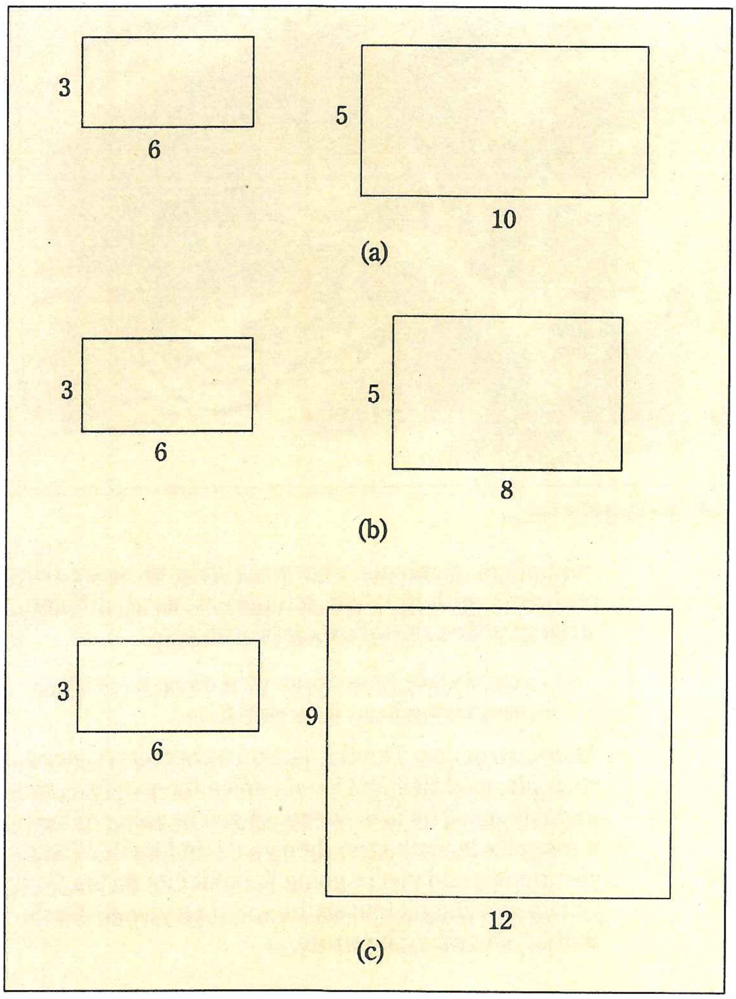


Fig. 6 Similar and nonsimilar rectangles

The rectangles in **figure 6c** are not similar because the length has been increased by a factor of 2, whereas the width has been increased by a factor of 3, or the sides of the larger rectangle are 6 units longer than the sides of the smaller rectangle. Extending the discussion of additive and multiplicative relationships to geometric figures will help students further develop the concepts of proportionality that are common to numeric and geometric contexts.

Alice's Adventures in Wonderland can provide a rich environment for mathematics and language learning. Reading the story while learning about topics such as multiplication of fractions, multiplicative change, proportional reasoning, or similarity provides a context of imaginative representation in which students can explore, discuss, and deepen their understanding of mathematics. Focusing on the mathematical aspects of the story will also help students analyze and gain a deeper appreciation for the literary qualities of the story.

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For more articles relating mathematics and literature, see the April 2005 focus issue of MTMS, *titled "Connecting Mathematics and Literature in the Middle Grades."—Ed.*

(See the worksheet on the next page.) □

How Tall Is Alice?

NAME ______________________

Show what happens to Alice's height when she eats cake or drinks potions.

A Cake That Makes Her **6 Times** as Tall		A Potion That Makes Her 1/6 as Tall	
Starting Height	Equation	Starting Height	Equation
4 feet		4 feet	
8 feet		8 feet	
2 feet		2 feet	
3/4 foot		3/4 foot	
3/8 foot		3/8 foot	

A Potion That Makes Her **5/3 Times** as Tall		A Cake That Makes Her 3/5 as Tall	
Starting Height	Equation	Starting Height	Equation
10 feet		10 feet	
12 feet		12 feet	
8 feet		8 feet	
2 feet		2 feet	
6 feet		6 feet	

If Alice drinks a potion that multiplies her height by 7/8, will she become taller or shorter? Why?

Make up stories for these problems:

1. $3/4 \times 27 =$

2. $15 \times 2/3 =$

3. $5/6 \times 20 =$

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